Five-Dimensional Brane World Theory

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A five-dimensional cosmological theory of gravitation that unifies space, time and velocity is presented. Within the framework of this theory we first discuss some general aspects of the universe in five dimensions. We then find the equations of motion of the expanding universe and show that it is accelerating. This followed by dealing with the important problem of halo dark matter around galaxies by deriving the equations of motion of a star moving around the field of a spherically-symmetric galaxy. The equations obtained are not Newtonian; rather, the Tully-Fisher formula is obtained. The cosmological constant is subsequently discussed: our theory predicts that $\Lambda \approx 3H_0^2 \approx 1.934 \times 10^{-35}\mathrm{s}^{-2}$, in agreement with experimental results obtained by the High-Z Supernova Team and the Supernova Cosmology Project. Finally we derive a formula for the cosmological redshift in which appears the expression $(1-\Omega_M)$, thus enabling us to determine the kind of the universe by means of the cosmological redshift. We find that Ω_M should be less than 1 in order not to contradict redshift measurements, and therefore the universe is open.

PACS numbers: 04.50.+h, 11.25.Mj, 11.27.+d, 98.80.Cq

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1 Introduction

In this paper we present a five-dimensional cosmological theory of space, time and velocity. The added extra dimension of velocity to the usual four-dimensional spacetime will be evident in the sequel. Before introducing the theory we have to deal, as usual, with coordinate systems in cosmology. Other important basic issues will be dealt later on.

1.1 Cosmic Coordinate Systems: The Hubble Transformation

We will use *cosmic coordinate systems* that fill up spacetime. Given one system x, there is another one x' that differs from the original one by a *Hubble transformation*

$$x' = x + t_1 v, \qquad t_1 = \text{constant}, \tag{1.1}$$

where v is a velocity parameter, and y and z are kept unchanged. A third system will be given by another Hubble transformation,

$$x'' = x' + t_2 v = x + (t_1 + t_2)v. \tag{1.2}$$

The cosmic coordinate systems are similar to the inertial coordinate systems, but now the velocity parameter takes over the time parameter and visa versa. The analogous Galileo transformation to Eq. (1.2) that relates inertial coordinate systems is given, as is known, by

$$x'' = x' + v_2 t = x + (v_1 + v_2)t. \tag{1.3}$$

The universe expansion is also given by a formula of the above kind:

$$x' = x + \tau v, \tag{1.4}$$

where $\tau = H_0^{-1}$ in the limit of zero distance, and thus a universal constant. (Its value is calculated in Subsection 5D as $\tau = 12.486$Gyr.) However, the universe expansion is apparently incompatible with the Hubble spacetime transformation, namely one cannot add them. Thus, if we have

$$x'' = x' + tv, \qquad x' = \tau v, \tag{1.5}$$

then

$$x'' \neq (\tau + t)\, v. \tag{1.6}$$

Rather, it is always

$$x'' = \tau v. \tag{1.7}$$

The above can be looked upon as a postulate of the theory.

This situation is like that we have with the propagation of light,

$$x'' \neq (c+v)\,t, \tag{1.8}$$

but it is always

$$x'' = ct \tag{1.9}$$

in all inertial coordinate systems, and where c is the speed of light in vacuum.

The constancy of the speed of light and the validity of the laws of nature in inertial coordinate systems, though they are both experimentally valid, they are apparently not compatible with each other. We have the same situation in cosmology; the constancy of the Hubble constant in the zero-distance limit, and the validity of the laws of nature in cosmic coordinate systems, though both are valid, they are apparently incompatible with each other.

1.2 Lorentz-like Cosmological Transformation

In the case of light propagation, one has to abandon the Galileo transformation in favor of the Lorentz transformation. In cosmology one has to give up the Hubble transformation in favor of a new Lorentz-like cosmological transformation given by [1]

$$x' = \frac{x - tv}{\sqrt{1 - t^2/\tau^2}}, \qquad v' = \frac{v - tx/\tau^2}{\sqrt{1 - t^2/\tau^2}}, \qquad y' = y, \qquad z' = z, \tag{1.10}$$

for a motion with fixed y and z.

As is well known, the flat spacetime line element in special relativity is given by

$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2). \tag{1.11}$$

The cosmological flat spacevelocity line element is given, accordingly, by

$$ds^2 = \tau^2dv^2 - (dx^2 + dy^2 + dz^2). \tag{1.12}$$

The special relativistic line element is invariant under the Lorentz transformation. So is the cosmological line element: it is invariant under the Lorentz-like cosmological transformation. The first keeps invariant the propagation of light, whereas the second keeps invariant the expansion of the universe. At small velocities with respect to the speed of light, $v \ll c$, the Lorentz transformation goes over to the nonrelativistic Galileo transformation. So is the situation in cosmology: the Lorentz-like cosmological transformation goes over to the nonrelativistic Hubble transformation (see Subsection A) that is valid for cosmic times much smaller than the Hubble time, $t \ll \tau$.

1.3 Five-Dimensional Manifold of Space, Time and Velocity

If we add the time to the cosmological flat spacevelocity line element, we obtain

$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2) + \tau^2 dv^2. \tag{1.13}$$

Accordingly, we have a five-dimensional manifold of time, space and velocity. The above line element provides a group of transformations O(2,3). At v=const it yields the Minkowskian line element (1.11); at t=const it gives the cosmological line element (1.12); and at a fixed space point, $dx = dy = dz = 0$, it leads to a new two-dimensional line element

$$ds^2 = c^2dt^2 + \tau^2 dv^2. \tag{1.14}$$

The groups associated with the above line elements are, of course, O(1,3), O(3,1) and O(2), respectively. They are the Lorentz group, the cosmological group and a two-dimensional Euclidean group, respectively.

In Section II we discuss some properties of the universe with gravitation in five dimensions. That includes the Bianchi identities, the gravitational field equations, the velocity as an independent coordinate and the enrgy density in cosmology. In Section III we find the equations of motion of the expanding universe and show that the universe is accelerating. In Section IV we discuss the important problem of halo dark matter around galaxies by finding the equations of motion of a star moving around a spherically-symmetric galaxy. The equations obtained are *not* Newtonian and instead the Tully-Fisher formula is obtained from our theory. In Section V the cosmological constant is discussed. Our theory predicts that $\Lambda \approx 3H_0^2 \approx 1.934 \times 10^{-35}\text{s}^{-2}$, in agreement with the supernovae experiments teams. In Section VI once again we show that the universe is infinite and open, now by applying redshift analysis, using a new formula that is derived here. Section VII is devoted to the concluding remarks.

2 Universe with Gravitation

The universe is, of course, not flat but filled up with gravity. When gravitation is invoked, the above spaces become curved Riemanian with the line element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \tag{2.1}$$

where μ, ν take the values 0, 1, 2, 3, 4. The coordinates are: $x^0 = ct$, x^1, x^2, x^3 are spatial coordinates and $x^4 = \tau v$ (the role of the velocity as an independent coordinate will be discussed in the sequel). The signature is $(+ - - - +)$. The metric tensor $g_{\mu\nu}$ is symmetric and thus we have fifteen independent components. They will be a solution of the Einstein field equations in five dimensions.

A discussion on the generalization of the Einstein field equations from four to five dimensions will also be given.

The five-dimensional field equations will not explicitely include a cosmological constant. Our cosmological constant, extracted from the theory, will be equal to $\Lambda = 3/\tau^2 = 1.934 \times 10^{-35}\text{s}^{-2}$ (for $H_0 = 73$km/s-Mpc). This should be compared with results of the experiments recently done with the supernovae which suggest the value of $\Lambda \approx 10^{-35}\text{s}^{-2}$. Our cosmological constant is derived from the theory itself which is necessary to the classification of the cosmological spaces to describe deccelerating, constant or accelerating universe. We now discuss some basic questions that are encountered in going from four to five dimensions.

2.1 The Bianchi Identities

The restricted Bianchi identities are given by [2]

$$\left(R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R\right)_{;\nu} = 0, \tag{2.2}$$

where μ,ν=0,...,4. They are valid in five dimensions just as they are in four dimensions. In Eq. (2.2) R_μ^ν and R are the Ricci tensor and scalar, respectively, and a semicolon denotes covariant differentiation. As a consequence we now have five coordinate conditions which permit us to determine five coordinates. For example, one can choose $g_{00} = 1$, $g_{0k} = 0$, $g_{44} = 1$, where k=1, 2, 3. These are the co-moving coordinates in five dimensions that keep the clocks and the velocity-measuring instruments synchronized. We will not use these coordinates in this paper.

2.2 The Gravitational Field Equations

In four dimensions these are the Einstein field equations [3]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \tag{2.3}$$

or equivalently

$$R_{\mu\nu} = \kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right), \tag{2.4}$$

where $T = g_{\alpha\beta}T^{\alpha\beta}$, and we have $R = -\kappa T$. In five dimensions if one chooses Eq. (2.3) as the field equations then Eq. (2.4) is *not* valid (the factor $\frac{1}{2}$ will have to be replaced by $\frac{1}{3}$, and $R = -\frac{2}{3}\kappa T$), and thus there is no symmetry between $-\kappa T$ and R.

In fact, if one assumes Eq. (2.4) to be valid in five dimensions then a simple calculation shows that $\kappa T = -\frac{2}{3}R$ and Eq. (2.4) becomes

$$R_{\mu\nu} - \frac{1}{3}g_{\mu\nu}R = \kappa T_{\mu\nu}. \tag{2.5}$$

Using now the Bianchi identities (2.2) then leads to $\partial R/\partial x^\nu = 0$ and thus $\partial T/\partial x^\nu = 0$. If one chooses, for example, the expression for the energy-momentum tensor to be given by $T^{\mu\nu} = \rho\,(dx^\mu/ds)\,(dx^\nu/ds)$, then $T = T^{\mu\nu} g_{\mu\nu} = \rho$. But if T is a constant then ρ is a constant (independent of time). This is obviously unacceptable situation for the universe since ρ decreases as the universe expands. Hence Eq. (2.4) has to be rejected on physical grounds, and the field equations that will be used by us in five dimensions are those given by (2.3).

Finally it is worthwhile mentioning that the only field equations in five dimensions that have symmetry between geometry and matter like those in the Einstein field equations in four dimensions are:

$$R_\mu^\nu - \frac{2}{5}\delta_\mu^\nu R = \kappa T_\mu^\nu, \tag{2.6}$$

$$R_\mu^\nu = \kappa\left(T_\mu^\nu - \frac{2}{5}\delta_\mu^\nu T\right), \tag{2.7}$$

with $R = -\kappa T$. While these equations are interesting, however, they do not reduce to Newtonian gravity and thus they do not seem to be of physical interest.

2.3 Velocity as an Independent Coordinate

First we have to iterate what do we mean by coordinates in general and how one measures them. The time coordinate is measured by clocks as was emphasized by Einstein repeatedly [4,5]. So are the spatial coordinates: they are measured by meters, as was originally done in special relativity theory by Einstein, or by use of Bondi's more modern version of k-calculus [6,7].

But how about the velocity as an independent coordinate? One might incline to think that if we know the spatial coordinates then the velocities are just the time derivative of the coordinates and they are not independent coordinates. This is, indeed, the situation for a dynamical system when the coordinates are given as functions of the time. But in general the situation is different, especially in cosmology. Take, for instance, the Hubble law $v = H_0 x$. Obviously v and x are independent parameters and v is not the time derivative of x. Basically one can measure v by instruments like those used by traffic police.

2.4 Effective Mass Density in Cosmology

To finish this section we discuss the important concept of the energy density in cosmology. We use the Einstein field equations, in which the right-hand side includes the energy-momentum tensor. For fields other than gravitation, like the electromagnetic field, this is a straightforward expression that comes out as a generalization to curved spacetime of the same tensor appearing in special-relativistic electrodynamics. However, when dealing with matter one should construct the energy-momentum tensor according to the physical situation (see,

for example, Fock, Ref. 26). Often a special expression for the mass density ρ is taken for the right-hand side of Einstein's equations, which sometimes is expressed as a δ-function.

In cosmology we also have the situation where the mass density is put on the right-hand side of the Einstein field equations. There is also the (constant) critical mass density $\rho_c = 3/8\pi G\tau^2$, the value of which is about 10^{-29} g/cm^3, just a few hydrogen atoms per cubic meter throughout the cosmos. If the universe average mass density ρ is equal to ρ_c then the three spatial geometry of the four-dimensional cosmological space is Euclidian. A deviation from this Euclidian geometry necessiates an increase or decrease of ρ_c. That is to say that

$$\rho_{eff} = \rho - \rho_c \tag{2.8}$$

is the active or the effective mass density that causes the three geometry not to be Euclidian. Accordingly, one should use ρ_{eff} in the right-hand side of the Einstein field equations. Indeed, we will use such a convention throughout this paper. The subtraction of ρ_c from ρ in not significant for celestial bodies amd makes no difference.

3 The Accelerating Universe

3.1 Preliminaries

In the last two sections we gave arguments to the fact that the universe should be presented in five dimensions, even though the standard cosmological theory is obtained from Einstein's four-dimensional general relativity theory. The situation here is similar to that prevailed before the advent of ordinary special relativity. At that time the equations of electrodynamics, written in three dimensions, were well known to predict that the speed of light was constant. But that was not the end of the road. The abandon of the concept of absolute space along with the constancy of the speed of light led to the four-dimensional notion. In cosmology now, we have to give up the notion of absolute cosmic time. Then this with the constancy of the Hubble constant in the limit of zero distance leads us to a five-dimensional presentation of cosmology.

We recall that the field equations are those of Einstein in five dimensions,

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = \kappa T_\mu^\nu, \tag{3.1}$$

where Greek letters $\alpha, \beta, \cdots, \mu, \nu, \cdots = 0, 1, 2, 3, 4$. The coordinates are $x^0 = ct$, x^1, x^2 and x^3 are space-like coordinates, $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$, and $x^4 = \tau v$.

The metric used is given in an approximate form by (see Appendix A)

$$g_{\mu\nu} = \begin{pmatrix} 1+\phi & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1+\psi \end{pmatrix}, \tag{3.2}$$

We will keep only linear terms. The nonvanishing Christoffel symbols are given by (see Appendix A)

$$\Gamma^0_{0\lambda} = \frac{1}{2}\phi_{,\lambda},\ \Gamma^0_{44} = -\frac{1}{2}\psi_{,0},\ \Gamma^n_{00} = \frac{1}{2}\phi_{,n},\ \Gamma^n_{44} = \frac{1}{2}\psi_{,n},\ \Gamma^4_{00} = -\frac{1}{2}\phi_{,4},\ \Gamma^4_{4\lambda} = \frac{1}{2}\psi_{,\lambda}, \tag{3.3}$$

where $n = 1, 2, 3$ and a comma denotes partial differentiation. The components of the Ricci tensor and the Ricci scalar are given by (Appendix B)

$$R^0_0 = \frac{1}{2}\left(\nabla^2\phi - \phi_{,44} - \psi_{,00}\right), \tag{3.4a}$$

$$R^n_0 = \frac{1}{2}\psi_{,0n}, \qquad R^0_n = -\frac{1}{2}\psi_{,0n}, \qquad R^4_0 = R^0_4 = 0, \tag{3.4b}$$

$$R^n_m = \frac{1}{2}\left(\phi_{,mn} + \psi_{,mn}\right), \tag{3.4c}$$

$$R^4_n = -\frac{1}{2}\phi_{,n4}, \qquad R^n_4 = \frac{1}{2}\phi_{,n4}. \tag{3.4d}$$

$$R^4_4 = \frac{1}{2}\left(\nabla^2\psi - \phi_{,44} - \psi_{,00}\right), \tag{3.4e}$$

$$R = \nabla^2\phi + \nabla^2\psi - \phi_{,44} - \psi_{,00}. \tag{3.5}$$

In the above equations ∇^2 is the ordinary Laplace operator.

3.2 Expanding Universe

The line element in five dimensions is given by

$$ds^2 = (1+\phi)dt^2 - dr^2 + (1+\psi)dv^2, \tag{3.6}$$

where $dr^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$, and where c and τ were taken, for brevity, as equal to 1. For an expanding universe one has $ds = 0$. The line element (3.6) represents a spherically symmetric universe.

The expansion of the universe (the Hubble expansion) is recorded at a definite fixed time and thus $dt = 0$. Accordingly Eq. (3.6) gives the following equation for the (spherically symmetric) expansion of the universe at a certain moment,

$$-dr^2 + (1+\psi)dv^2 = 0, \tag{3.7}$$

and thus

$$\left(\frac{dr}{dv}\right)^2 = 1 + \psi. \tag{3.8}$$

To find ψ we solve the Einstein field equation (noting that $T_0^0 = g_{0\alpha}T^{\alpha 0} \approx T^{00} = \rho(dx^0/ds)^2 \approx c^2\rho$, or $T_0^0 \approx \rho$ in units with $c = 1$):

$$R_0^0 - \frac{1}{2}\delta_0^0 R = 8\pi G\rho_{eff} = 8\pi G\left(\rho - \rho_c\right), \tag{3.9}$$

where $\rho_c = 3/8\pi G\tau^2$.

A simple calculation using Eqs. (3.4a) and (3.5) then yields

$$\nabla^2\psi = 6(1 - \Omega_M), \tag{3.10}$$

where $\Omega_M = \rho/\rho_c$.

The solution of the field equation (3.10) is given by

$$\psi = (1 - \Omega_M)r^2 + \psi_0, \tag{3.11}$$

where the first part on the right-hand side is a solution for the non-homogeneous Eq. (3.10), and ψ_0 represents a solution to its homogeneous part, i.e. $\nabla^2\psi_0 = 0$. A solution for ψ_0 can be obtained as an infinite series in powers of r. The only term that is left is of the form $\psi_0 = -K_2/r$, where K_2 is a constant whose value can easily be shown to be the Schwartzschild radius, $K_2 = 2GM$. We therefore have

$$\psi = (1 - \Omega_M)r^2 - 2GM/r. \tag{3.12}$$

The universe expansion is therefore given by

$$\left(\frac{dr}{dv}\right)^2 = 1 + \left(1 - \Omega_M\right)r^2 - \frac{2GM}{r}. \tag{3.13}$$

For large r the last term on the right-hand side of (3.13) can be neglected, and therefore

$$\left(\frac{dr}{dv}\right)^2 = 1 + (1 - \Omega_M)r^2, \tag{3.14}$$

or

$$\frac{dr}{dv} = \left[1 + \left(1 - \Omega_M\right)r^2\right]^{1/2}. \tag{3.15}$$

Inserting now the constants c and τ we finally obtain for the expansion of the universe

$$\frac{dr}{dv} = \tau\left[1 + \left(1 - \Omega_M\right)r^2/c^2\tau^2\right]^{1/2}. \tag{3.16}$$

This result is exactly that obtained by Behar and Carmeli (BC) (Eq. 5.10) when the non-relativistic relation $z = v/c$, where z is the redshift parameter, is inserted in the previous result [8].

The second term in the square brackets of (3.16) represents the deviation from constant expansion due to gravity. For without this term, Eq. (3.16) reduces to $dr/dv = \tau$, thus $r = \tau v$+const. The constant can be taken zero if one assumes, as usual, that at $r = 0$ the velocity should also vanish. Accordingly we have $r = \tau v$ or $v = \tau^{-1} r$. Hence when $\Omega_M = 1$, namely when $\rho = \rho_c$, we have a constant expansion.

3.3 Decelerating, Constant and Accelerating Expansions

The equation of motion (3.16) can be integrated exactly (see Appendix C). The results are:
For the $\Omega_M > 1$ case

$$r(v) = (c\tau/\alpha)\sin(\alpha v/c)\,; \qquad \alpha = (\Omega_M - 1)^{1/2}\,. \tag{3.17}$$

This is obviously a decelerating expansion.
For $\Omega_M < 1$,

$$r(v) = (c\tau/\beta)\sinh(\beta v/c)\,; \qquad \beta = (1 - \Omega_M)^{1/2}\,. \tag{3.18}$$

This is now an accelerating expansion.

For $\Omega_M = 1$ we have, from Eq. (3.16),

$$d^2r/dv^2 = 0, \tag{3.19}$$

whose solution is, of course,

$$r(v) = \tau v, \tag{3.20}$$

and this is a constant expansion. It will be noted that the last solution can also be obtained directly from the previous two cases for $\Omega_M > 1$ and $\Omega_M < 1$ by going to the limit $v \to 0$, using L'Hospital's lemma, showing that our solutions are consistent.

It has been shown in BC that the constant expansion is just a transition stage between the decelerating and the accelerating expansions as the universe evolves toward its present situation. That occured at about 5Gyr from the Big Bang at a time the cosmic radiation temperature was 146K [8].

3.4 Accelerating Universe

In order to decide which of the three cases is the appropriate one at the present time, it will be convenient to write the solutions (3.17), (3.18) and (3.20) in the ordinary Hubble law form $v = H_0 r$. Expanding Eqs. (3.17) and (3.18) and keeping the appropriate terms then yields

$$r = \tau v\left(1 - \alpha^2 v^2/6c^2\right), \tag{3.21}$$

$$r = \tau v \left(1 + \beta^2 v^2 / 6c^2\right), \tag{3.22}$$

for the $\Omega_M > 1$ and $\Omega_M < 1$ cases, respectively. Using now the expressions for α and β in Eqs. (3.21) and (3.22), then both of the latter can be reduced into the single equation

$$r = \tau v \left[1 + (1 - \Omega_M) v^2 / 6c^2\right]. \tag{3.23}$$

Inverting now this equation by writing it in the form $v = H_0 r$, we obtain in the lowest approximation for H_0

$$H_0 = h \left[1 - (1 - \Omega_M) v^2 / 6c^2\right], \tag{3.24}$$

where $h = 1/\tau$. Using $v \approx r/\tau$, or $z \approx v/c$, we also obtain

$$H_0 = h \left[1 - (1 - \Omega_M) r^2 / 6c^2\tau^2\right] = h \left[1 - (1 - \Omega_M) z^2 / 6\right]. \tag{3.25}$$

As is seen H_0 depends on the distance, or equivalently, on the redshift. Cosequently H_0 has meaning only in the limits $r \to 0$ and $z \to 0$, namely when measured *locally*, in which case it becomes the constant h. This is similar to the situation with respect to the speed of light when measured globally in the presence of gravitational field as the ratio between distance and time, the result usually depends on these parameters. Only in the limit one obtains the constant speed of light in vacuum ($c \approx 3 \times 10^{10}$cm/s).

As is seen from the above discussion, H_0 is intimately related to the sign of the factor $(1 - \Omega_M)$. If measurements of H_0 indicate that it increases with the redshift parameter z then the sign of $(1 - \Omega_M)$ is negative, namely $\Omega_M > 1$. If, however, H_0 decreases when z increases then the sign of $(1 - \Omega_M)$ is positive, i.e. $\Omega_M < 1$. The possibility of H_0 not to depend on the redshift parameter indicates that $\Omega_M = 1$. In recent years different measurements were obtained for H_0, with the so-called "short" and "long" distance scales, in which higher values of H_0 were obtained for the short distances and the lower values for H_0 corresponded to the long distances [9-18]. Indications are that the longer the distance of measurement, the smaller the value of H_0. If one takes these experimental results seriously, then that is possible only for the case in which $\Omega_M < 1$, namely when the universe is at an accelerating expansion phase, and the universe is thus open. We will see in Section VI that the same result is obtained via a new cosmological redshift formula.

4 The Tully-Fisher Formula: Nonexistence of Halo Dark Matter?

In this section we derive the equations of motion of a star moving around a spherically symmetric galaxy and show that the Tully-Fisher formula is obtained from our five-dimensional cosmological theory. The calculation is lengthy but it is straightforward. The equations of motion will first be of general nature and only

afterward specialized to the motion of a star around the field of a galaxy. The equations obtained are *not* Newtonian. The Tully-Fisher formula was obtained by us in a previous paper [19] using two representations of Einstein's general relativity: the standard spacetime theory and a spacevelocity version of it. However, the present derivation is a straightforward result from the unification of space, time and velocity.

Our notation in this section is as follows: $\alpha, \beta, \gamma, \ldots = 0, \cdots, 4$; $a, b, c, d, \cdots = 0, \cdots, 3$; $p, q, r, s, \cdots = 1, \cdots, 4$; and $k, l, m, n, \cdots = 1, 2, 3$. The coordinates are: $x^0 = ct$ (timelike), $x^k = x^1, x^2, x^3$ (spacelike), and $x^4 = \tau v$ (velocitylike – see Section 2C).

4.1 The Geodesic Equation

As usual the equations of motion are obtained in general relativity theory from the covariant conservation law of the energy-momentum tensor (which is a consequence of the restricted Bianchi identities), and the result, as is well known, is the geodesic equation that describes the motion of a spherically symmetric test particle. In our five-dimensional cosmological model we have five equations of motion. They are given by

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0. \tag{4.1}$$

We now change the independent parameter s into an arbitrary new parameter σ, then the geodesic equation becomes

$$\frac{d^2x^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = -\frac{d^2\sigma/ds^2}{\left(d\sigma/ds\right)^2}\frac{dx^\mu}{d\sigma}. \tag{4.2}$$

The parameter σ will be taken once as $\sigma = x^0$ (the time coordinate) and then $\sigma = x^4$ (the velocity coordinate). We obtain, for the first case,

$$\frac{d^2x^\mu}{\left(dx^0\right)^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dx^0}\frac{dx^\beta}{dx^0} = -\frac{d^2x^0/ds^2}{\left(dx^0/ds\right)^2}\frac{dx^\mu}{dx^0}. \tag{4.3}$$

The right-hand side of Eq. (4.3) can be written in a somewhat different form by using its zero component

$$\frac{d^2x^0}{\left(dx^0\right)^2} + \Gamma^0_{\alpha\beta}\frac{dx^\alpha}{dx^0}\frac{dx^\beta}{dx^0} = -\frac{d^2x^0/ds^2}{\left(dx^0/ds\right)^2}\frac{dx^0}{dx^0}. \tag{4.4}$$

But $dx^0/dx^0 = 1$, and $d^2x^0/(dx^0)^2 = 0$. Hence we obtain

$$\frac{d^2x^0/ds^2}{\left(dx^0/ds\right)^2} = -\Gamma^0_{\alpha\beta}\frac{dx^\alpha}{dx^0}\frac{dx^\beta}{dx^0}. \tag{4.5}$$

Using the above result in Eq. (4.3), the latter can be writtem in the form

$$\frac{d^2x^\mu}{\left(dx^0\right)^2}+\left(\Gamma^\mu_{\alpha\beta}-\Gamma^0_{\alpha\beta}\frac{dx^\mu}{dx^0}\right)\frac{dx^\alpha}{dx^0}\frac{dx^\beta}{dx^0}=0. \tag{4.6}$$

It will be noted that the zero component Eq. (4.6) is now an identity, and consequently it reduces to the four-dimensional equation

$$\frac{d^2x^p}{\left(dx^0\right)^2}+\left(\Gamma^p_{\alpha\beta}-\Gamma^0_{\alpha\beta}\frac{dx^p}{dx^0}\right)\frac{dx^\alpha}{dx^0}\frac{dx^\beta}{dx^0}=0, \tag{4.7}$$

where $p=1,2,3,4$.

In exactly the same way we parametrize the geodesic equation (4.1) now with respect to the velocity by choosing the parameter $\sigma=\tau v$. The result is

$$\frac{d^2x^a}{\left(dx^4\right)^2}+\left(\Gamma^a_{\alpha\beta}-\Gamma^4_{\alpha\beta}\frac{dx^a}{dx^4}\right)\frac{dx^\alpha}{dx^4}\frac{dx^\beta}{dx^4}=0, \tag{4.8}$$

where $a=0,1,2,3$.

The equation of motion (4.7) will be expanded in terms of the parameter v/c, assuming $v\ll c$, whereas Eq. (4.8) will be expanded with t/τ, where t is a characteristic cosmic time, and $t\ll\tau$. We then can use the Einstein-Infeld-Hoffmann (EIH) method that is well known in general relativity in obtaining the equations of motion [20-37].

We start with Eq. (4.7). As is seen the second term in the paranthesis can be neglected with respect to the first one since $d/dx^0=(1/c)d/dt$, and we obtain

$$\frac{d^2x^p}{\left(dx^0\right)^2}+\Gamma^p_{\alpha\beta}\frac{dx^\alpha}{dx^0}\frac{dx^\beta}{dx^0}=0. \tag{4.9}$$

In Eq. (4.8) we also can neglect the second term in the paranthesis since $d/dx^4=(1/\tau)d/dv$. As a result we have the approximate equations of motion

$$\frac{d^2x^a}{\left(dx^4\right)^2}+\Gamma^a_{\alpha\beta}\frac{dx^\alpha}{dx^4}\frac{dx^\beta}{dx^4}=0. \tag{4.10}$$

The equations of motion (4.9) and (4.10) are consequently given by

$$\frac{d^2x^p}{dt^2}+\Gamma^p_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}=0, \tag{4.11}$$

$$\frac{d^2x^a}{dv^2}+\Gamma^a_{\alpha\beta}\frac{dx^\alpha}{dv}\frac{dx^\beta}{dv}=0. \tag{4.12}$$

To find the lowest approximation of Eq. (4.11), since $dx^0/dt\gg dx^q/dt$, all terms with indices that are not zero-zero can be neglected. Consequently, Eq. (4.11) is reduced to the form

$$\frac{d^2x^p}{dt^2}\approx-\Gamma^p_{00}, \tag{4.13}$$

in the lowest approximation.

4.2 Equations of Motion

Accordingly Γ^p_{00} acts like a Newtonian force per mass unit. In terms of the metric tensor we therefore obtain, since $\Gamma^p_{00} = -\frac{1}{2}\eta^{pq}\phi_{,q}$ (see Appendix A)

$$\frac{d^2x^p}{dt^2} \approx -\frac{1}{2}\eta^{pq}\frac{\partial\phi}{\partial q}, \tag{4.14}$$

where $\phi = g_{00} - 1$ (see Appendix A). We now decompose this equation into a spatial ($p = 1, 2, 3$) and a velocity ($p = 4$) parts, getting

$$\frac{d^2x^k}{dt^2} = -\frac{1}{2}\frac{\partial\phi}{\partial x^k}, \tag{4.15a}$$

$$\frac{d^2v}{dt^2} = 0. \tag{4.15b}$$

Using exactly the same method, Eq. (4.12) yields

$$\frac{d^2x^k}{dv^2} = -\frac{1}{2}\frac{\partial\psi}{\partial x^k}, \tag{4.16a}$$

$$\frac{d^2t}{dv^2} = 0, \tag{4.16b}$$

where $\psi = g_{44}-1$. In the above equations $k = 1, 2, 3$. Equation (4.15a) is exactly the law of motion with the function ϕ being twice the Newtonian potential. The other three equations Eq. (4.15b) and Eqs. (4.16a,b) are not Newtonian and are obtained only in the present theory. It remains to find out the functions ϕ and ψ.

To find out the function ϕ we solve the Einstein field equation (noting that $T^4_4 = g_{4\alpha}T^{\alpha 4} \approx T^{44} = \rho(dx^4/ds)^2 \approx \tau^2\rho$, and thus $T^4_4 \approx \rho$ in units in which $\tau = 1$):

$$R^4_4 - \frac{1}{2}\delta^4_4 R = 8\pi G\rho_{eff} = 8\pi G(\rho - \rho_c). \tag{4.17}$$

A straightforward calculation then gives

$$\nabla^2\phi = 6\left(1 - \Omega_M\right), \tag{4.18}$$

whose solution is given by

$$\phi = \left(1 - \Omega_M\right) r^2 + \phi_0, \tag{4.19}$$

where ϕ_0 is a solution of the homogeneous equation $\nabla^2\phi_0 = 0$. One then easily finds that $\phi_0 = -K_1/r$, where $K_1 = 2GM$. In the same way the function ψ can be found (see Section 3),

$$\psi = \left(1 - \Omega_M\right) r^2 + \psi_0, \tag{4.20}$$

with $\nabla^2\psi_0 = 0$, $\psi_0 = -K_2/r$ and $K_2 = 2GM$. (When units are inserted then $K_1 = 2GM/c^2$ and $K_2 = 2GM\tau^2/c^2$.) For the purpose of obtaining equations of motion one can neglect the terms $(1-\Omega_M)r^2$, actually $(1-\Omega_M)r^2/c^2\tau^2$, in the solutions for ϕ and ψ. One then obtains

$$g_{00} \approx 1 - 2GM/c^2 r, \qquad g_{44} \approx 1 - 2GM\tau^2/c^2 r. \tag{4.21}$$

The equations of motion (4.15a) and (4.16a), consequently, have the forms

$$\frac{d^2x^k}{dt^2} = \left(\frac{GM}{r}\right)_{,k}, \qquad \frac{d^2x^k}{dv^2} = \left(\frac{GM}{r}\right)_{,k}, \tag{4.22}$$

or, when inserting the constants c and τ,

$$\frac{d^2x^k}{dt^2} = GM\left(\frac{1}{r}\right)_{,k}, \tag{4.23a}$$

$$\frac{d^2x^k}{dv^2} = kM\left(\frac{1}{r}\right)_{,k}, \tag{4.23b}$$

where $k = G\tau^2/c^2$. It remains to integrate equations (4.15b) and (4.16b). One finds that $v = a_0 t$, where a_0 is a constant which can be taken as equal to $a_0 = c/\tau \approx cH_0$. Accordingly, we see that the particle experiences an acceleration $a_0 = c/\tau \approx cH_0$ directed outward when the motion is circular.

Equation (4.23a) is Newtonian but (4.23b) is not. The integration of the latter is identical to that familiar in classical Newtonian mechanics, but there is an essential difference which should be emphasized. In Newtonian equations of motion one deals with a path of motion in the 3-space. In our theory we do not have that situation. Rather, the paths here indicate locations of particles in the sense of the Hubble distribution, which now takes a different physical meaning. With that in mind we proceed as follows.

Equation (4.23b) yields the first integral

$$\left(\frac{ds}{dv}\right)^2 = \frac{kM}{r}, \tag{4.24a}$$

where v is the velocity of the particles, in analogy to the Newtonian

$$\left(\frac{ds}{dt}\right)^2 = \frac{GM}{r}, \tag{4.24b}$$

In these equations s is the length parameter along the path of the accumulation of the particles.

Comparing Eqs. (4.24a) and (4.24b), we obtain

$$\frac{ds}{dv} = \frac{\tau}{c}\frac{ds}{dt}. \tag{4.25}$$

Thus

$$\frac{dv}{dt} = \frac{c}{\tau}. \tag{4.26}$$

Accordingly, as we have mentioned before, the particle experiences an acceleration $a_0 = c/\tau \approx cH_0$ directed outward when the motion is circular.

4.3 The Tully-Fisher Law

The motion of a particle in a central field is best described in terms of an "effective potential", V_{eff}. In Newtonian mechanics this is given by [38]

$$V_{eff} = -\frac{GM}{r} + \frac{L^2}{2r^2}, \tag{4.27}$$

where L is the angular momentum per mass unit. In our case the effective potential is

$$V_{eff}(r) = -\frac{GM}{r} + \frac{L^2}{2r^2} + a_0 r. \tag{4.28}$$

The circular motion is obtained at the minimal value of (4.28), i.e.,

$$\frac{dV_{eff}}{dr} = \frac{GM}{r^2} - \frac{L^2}{r^3} + a_0 = 0, \tag{4.29}$$

with $L = v_c r$, and v_c is the circular velocity. This gives

$$v_c^2 = \frac{GM}{r} + a_0 r. \tag{4.30}$$

Thus

$$v_c^4 = \left(\frac{GM}{r}\right)^2 + 2GMa_0 + a_0^2 r^2, \tag{4.31}$$

where $a_0 = c/\tau \approx cH_0$.

The first term on the right-hand side of Eq. (4.31) is purely Newtonian, and cannot be avoided by any reasonable theory. The second one is the Tully-Fisher term. The third term is extremely small at the range of distances of stars around a galaxy. It is well known that astronomical observations show that for disk galaxies the fourth power of the circular velocity of stars moving around the core of the galaxy, v_c^4, is proportional to the total luminosity L of the galaxy to an occuracy of more than two orders of magnitude in L, namely $v_c^4 \propto L$ [39]. Since L is proportional to the mass M of the galaxy, one obtains $v_c^4 \propto M$. This is the Tully-Fisher law. There is no dependence on the distance of the star from the center of the galaxy as Newton's law $v_c^2 = GM/r$ requires for circular motion. In order to rectify this deviation from Newton's laws, astronomers assume the existence of halos around the galaxy which are filled with dark matter and arranged in such a way so as to satisfy the Tully-Fisher law for each particular situation.

In conclusion it appears that there is no necessity for the assumption of the existence of halo dark matter around galaxies. Rather, the result can be described in terms of the properties of spacetimevelocity.

5 The Cosmological Constant

5.1 The Cosmological Term

First, a historical remark. In order to allow the existence of a static solution for the gravitational field equations, Einstein made a modification to his original equations (2.3) by adding a cosmological term,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \tag{5.1}$$

where Λ is the cosmological constant and $\kappa = 8\pi G$. For a homogeneous and isotropic universe with the line element [40,41]

$$ds^2 = dt^2 - a^2(t) R_0^2 \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right], \tag{5.2}$$

where k is the curvature parameter ($k = 1, 0, -1$) and $a(t) = R(t)/R_0$ is the scale factor, with the energy-momentum tensor

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}, \tag{5.3}$$

Einstein's equations (5.1) reduce to the two Friedmann equations

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{\kappa}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2 R_0^2}, \tag{5.4}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6} (\rho + 3p) + \frac{\Lambda}{3}. \tag{5.5}$$

These equations admit a static solution ($\dot{a} = 0$) with $k > 0$ and $\Lambda > 0$. After Hubble's discovery that the universe is expanding, the role of the cosmological constant to allow static homogeneous solutions to Einstein's equations in the presence of matter, seemed to be unnecessary. For a long time the cosmological term was considered to be of no physical interest in cosmological problems.

From the Friedmann equation (5.4), for any value of the Hubble parameter H there is a critical value of the mass density such that the spatial geometry is flat ($k = 0$), $\rho_c = 3H_0^2/\kappa$ (see Subsection IID). One usually measures the total mass density in terms of the critical density ρ_c by means of the density parameter $\Omega_M = \rho/\rho_c$.

In general, the mass density ρ includes contributions from various distinct components. From the point of view of cosmology, the relevant aspect of each

component is how its energy density evolves as the universe expands. In general, a positive Λ causes acceleration to the universe expansion, whereas a negative Λ and ordinary matter tend to decelerate it. Moreover, the relative contributions of the components to the energy density change with time. For $\Omega_\Lambda < 0$, the universe will always recollapse to a Big Crunch. For $\Omega_\Lambda > 0$ the universe will expand forever unless there is sufficient matter to cause recollapse before Ω_Λ becomes dynamically important. For $\Omega_\Lambda = 0$ we have the familiar situation in which $\Omega_M \leq 1$ universes expand forever and $\Omega_M > 1$ universes recollapse. (For more details see the paper by Behar and Carmeli, Ref. 8.)

5.2 The Supernovae Experiments Value for Λ

Recently two groups, the *Supernova Cosmology Project Collaboration* and the *High-Z Supernova Team Collaboration*, presented evidence that the expansion of the universe is accelerating [42-48]. These teams have measured the distances to cosmological supernovae by using the fact that the intrinsic luminosity of Type Ia supernovae is closely correlated with their decline rate from maximum brightness, which can be independently measured. These measurements, combined with redshift data for the supernovae, led to the prediction of an accelerating universe. Both teams obtained

$$\Omega_M \approx 0.3, \qquad \Omega_\Lambda \approx 0.7, \tag{5.6}$$

and strongly ruled out the traditional $(\Omega_M, \Omega_\Lambda)$=(1, 0) universe. This value of the density parameter Ω_Λ corresponds to a cosmological constant that is small but nonzero and positive,

$$\Lambda \approx 10^{-52}\mathrm{m}^{-2} \approx 10^{-35}\mathrm{s}^{-2}. \tag{5.7}$$

5.3 The Behar-Carmeli Predicted Value for Λ

In the paper of Behar and Carmeli a four-dimensional cosmological relativity theory that unifies space and velocity was proposed that predicts the acceleration of the universe and hence it is equivalent to having a positive value for Λ in it. As is well known, in the traditional work of Friedmann when added to it a cosmological constant, the field equations obtained are highly complicated and no solutions have been obtained so far. Behar-Carmeli's theory, on the other hand, yields exact solutions and describes the universe as having a three-phase evolution with a decelerating expansion followed by a constant and an accelerating expansion, and it predicts that the universe is now in the latter phase. In the framework of this theory the zero-zero component of Einstein's equations is written as

$$R_0^0 - \frac{1}{2}\delta_0^0 R = \kappa\rho_{eff} = \kappa\left(\rho - \rho_c\right), \tag{5.8}$$

where $\rho_c = 3/\kappa\tau^2 \approx 3H^2/\kappa$ is the critical mass density. Comparing Eq. (5.8) with the zero-zero component of Eq. (5.1), one obtains the expression for the cosmological constant in the Behar-Carmeli theory,

$$\Lambda = \kappa\rho_c = 3/\tau^2 \approx 3H^2. \tag{5.9}$$

5.4 Comparison with Experiment

To find out the numerical value of τ we use the relationship between $h = \tau^{-1}$ and H_0 given by Eq. (3.25) (CR denote values according to Cosmological Relativity):

$$H_0 = h\left[1 - \left(1 - \Omega_m^{CR}\right) z^2/6\right], \tag{5.10}$$

where $z = v/c$ is the redshift and $\Omega_m^{CR} = \rho_M/\rho_c$ with $\rho_c = 3h^2/8\pi G$. (Notice that our $\rho_c = 1.194 \times 10^{-29} g/cm^3$ is different from the standard ρ_c defined with H_0.) The redshift parameter z determines the distance at which H_0 is measured. We choose $z = 1$ and take for

$$\Omega_m^{CR} = 0.245, \tag{5.11}$$

its value at the present time (corresponds to 0.32 in the standard theory), Eq. (5.10) then gives

$$H_0 = 0.874h. \tag{5.12}$$

At the value $z = 1$ the corresponding Hubble parameter H_0 according to the latest results from HST can be taken [9] as $H_0 = 70$km/s-Mpc, thus $h = (70/0.874)$km/s-Mpc, or

$$h = 80.092\text{km/s-Mpc}, \tag{5.13}$$

and

$$\tau = 12.486Gyr = 3.938 \times 10^{17}s. \tag{5.14}$$

What is left is to find the value of Ω_Λ^{CR}. We have $\Omega_\Lambda^{CR} = \rho_c^{ST}/\rho_c$, where $\rho_c^{ST} = 3H_0^2/8\pi G$ and $\rho_c = 3h^2/8\pi G$. Thus $\Omega_\Lambda^{CR} = (H_0/h)^2 = 0.874^2$, or

$$\Omega_\Lambda^{CR} = 0.764. \tag{5.15}$$

As is seen from Eqs. (5.11) and (5.15) one has

$$\Omega_T = \Omega_m^{CR} + \Omega_\Lambda^{CR} = 0.245 + 0.764 = 1.009 \approx 1, \tag{5.16}$$

which means the universe is Euclidean.

As a final result we calculate the cosmological constant according to Eq. (5.9). One obtains

$$\Lambda = 3/\tau^2 = 1.934 \times 10^{-35}s^{-2}. \tag{5.17}$$

Our results confirm those of the supernovae experiments and indicate on the existance of the dark energy as has recently received confirmation from the Boomerang cosmic microwave background experiment [50,51], which showed that the universe is Euclidean.

6 Cosmological Redshift Analysis

6.1 The Redshift Formula

In this section we derive a general formula for the redshift in which the term $(1-\Omega_M)$ appears explicitly. Since there are enough data of measurements of redshifts, this allows one to determine what is the sign of $(1-\Omega_M)$, positive, zero or negative. Our conclusion is that $(1-\Omega_M)$ cannot be negative or zero. This means that the universe is infinite, and expands forever, a result favored by some cosmologists [49]. To this end we proceed as follows.

Having the metric tensor from Section IV we may now find the redshift of light emitted in the cosmos. As usual, at two points 1 and 2 we have for the wave lengths and frequencies:

$$\frac{\lambda_2}{\lambda_1} = \frac{\nu_1}{\nu_2} = \frac{ds\,(2)}{ds\,(1)} = \sqrt{\frac{g_{00}\,(2)}{g_{00}\,(1)}}. \tag{6.1}$$

Using now the solution for $g_{00} = 1 + \phi$, with ϕ given by Eq. (4.19), in Eq. (6.1), we obtain

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{1 + r_2^2/a^2 - R_s/r_2}{1 + r_1^2/a^2 - R_s/r_1}}. \tag{6.2}$$

In Eq. (6.2) $R_s = 2GM/c^2$ and $a^2 = c^2\tau^2/(1-\Omega_M)$.

For a sun-like body with radius R located at the coordinates origin, and an observer at a distance r from the center of the body, we then have $r_2 = r$ and $r_1 = R$, thus

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{1 + r^2/a^2 - R_s/r}{1 + R^2/a^2 - R_s/R}}. \tag{6.3}$$

6.2 Particular Cases

Since $R \ll r$ and $R_s < R$ is usually the case we can write, to a good approximation,

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{1 + r^2/a^2}{1 - R_s/R}}. \tag{6.4}$$

The term r^2/a^2 in Eq. (6.4) is a pure cosmological one, whereas R_s/R is the standard general relativistic term. For $R \gg R_s$ we then have

$$\frac{\lambda_2}{\lambda_1} = \sqrt{1 + \frac{r^2}{a^2}} = \sqrt{1 + \frac{(1-\Omega_M)\,r^2}{c^2\tau^2}} \tag{6.5}$$

for the pure cosmological contribution to the redshift. If, furthermore, $r \ll a$ we then have

$$\frac{\lambda_2}{\lambda_1} = 1 + \frac{r^2}{2a^2} = 1 + \frac{(1-\Omega_M)\,r^2}{2c^2\tau^2} \tag{6.6}$$

to the lowest approximation in r^2/a^2, and thus

$$z = \frac{\lambda_2}{\lambda_1} - 1 = \frac{r^2}{2a^2} = \frac{(1-\Omega_M)\, r^2}{2c^2\tau^2}. \tag{6.7}$$

When the contribution of the cosmological term r^2/a^2 is negligible, we then have

$$\frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{1 - R_s/R}}. \tag{6.8}$$

The redshift could then be very large if R, the radius of the emitting body, is just a bit larger than the Schwarzchild radius R_s. For example if $R_s/R = 0.96$ the redshift $z = 4$. For a typical sun like ours, $R_s \ll R$ and we can expand the righthand side of Eq. (6.8), getting

$$\frac{\lambda_2}{\lambda_1} = 1 + \frac{R_s}{2R}, \tag{6.9}$$

thus

$$z = \frac{R_s}{2R} = \frac{Gm}{c^2 R}, \tag{6.10}$$

the standard general relativistic result.

From Eqs. (6.5)–(6.7) it is clear that Ω_M cannot be larger than one since otherwise z will be negative, which means blueshift, and as is well known nobody sees such a thing. If $\Omega_M = 1$ then $z = 0$, and for $\Omega_M < 1$ we have $z > 0$. The case of $\Omega_M = 1$ is also implausible since the light from stars we see is usually redshifted more than the redshift due to the gravity of the body emitting the radiation, as is evident from our sun, for example, whose emitted light is shifted by only $z = 2.12 \times 10^{-16}$ [3].

6.3 Conclusions

One can thus conclude that the theory of unified space, time and velocity predicts that the universe is open. As is well known the standard FRW model does not relate the cosmological redshift to the kind of universe.

7 Concluding Remarks

The most direct evidence that the universe expansion is accelerating and propelled by "dark energy", is provided by the faintness of Type Ia supernovae (SNe Ia) at $z \approx 0.5$ [42,46]. Beyond the redshift range of $0.5 < z < 1$, the universe was more compact and the attraction of matter was stronger than the repulsive dark energy. At $z > 1$ the expansion of the universe should have been decelerating [52]. At $z \geq 1$ one would expect an apparent brightness increase of SNe Ia relative to what is supposed to be for a non-decelerating universe [53].

Recently, more confirmation to the universe accelerating expansion came from the most distant supernova, SN 1997ff, that was recorded by the Hubble Space Telescope. As has been pointed out before, if we look back far enough, we should find a decelerating expansion. Beyond $z = 1$ one should see an earlier time when the mass density was dominant. The measurements obtained from SN 1997ff's redshift and brightness provide a direct proof for the transition from past decelerating to present accelerating expansion. The measurements also exclude the possibility that the acceleration of the universe is not real but is due to other astrophysical effects such as dust.

Appendix A: Mathematical Conventions and Christoffel Symbols

Throughout this paper we use the convention

$$\alpha, \beta, \gamma, \delta, \cdots = 0, 1, 2, 3, 4,$$

$$a, b, c, d, \cdots = 0, 1, 2, 3,$$

$$p, q, r, s, \cdots = 1, 2, 3, 4,$$

$$k, l, m, n, \cdots = 1, 2, 3.$$

The coordinates are $x^0 = ct$, x^1, x^2 and x^3 are space-like coordinates, $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$, and $x^4 = \tau v$. The signature is $(+ - - - +)$. The metric, approximated up to ϕ and ψ, is:

$$g_{\mu\nu} = \begin{pmatrix} 1+\phi & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1+\psi \end{pmatrix}, \tag{A.1}$$

$$g^{\mu\nu} = \begin{pmatrix} 1-\phi & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1-\psi \end{pmatrix}. \tag{A.2}$$

The nonvanishing Christoffel symbols are (in the linear approximation):

$$\Gamma^0_{0\lambda} = \frac{1}{2}\phi_{,\lambda}, \quad \Gamma^0_{44} = -\frac{1}{2}\psi_{,0}, \quad \Gamma^n_{00} = \frac{1}{2}\phi_{,n}, \tag{A.3a}$$

$$\Gamma^n_{44} = \frac{1}{2}\psi_{,n}, \quad \Gamma^4_{00} = -\frac{1}{2}\phi_{,4}, \quad \Gamma^4_{4\lambda} = \frac{1}{2}\psi_{,\lambda}, \tag{A.3b}$$

$$\Gamma^a_{00} = -\frac{1}{2}\eta^{ab}\phi_{,b}, \qquad \Gamma^a_{44} = -\frac{1}{2}\eta^{ab}\psi_{,b}, \tag{A.3c}$$

$$\Gamma^p_{00} = -\frac{1}{2}\eta^{pq}\phi_{,q}, \qquad \Gamma^p_{44} = -\frac{1}{2}\eta^{pq}\psi_{,q}. \tag{A.3d}$$

The Minkowskian metric η in five dimensions is given by

$$\eta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{A.4}$$

Appendix B: Components of the Ricci Tensor

The elements of the Ricci tensor are:

$$R_{00} = \frac{1}{2}\left(\nabla^2\phi - \phi_{,44} - \psi_{,00}\right), \tag{B.1}$$

$$R_{0n} = -\frac{1}{2}\psi_{,0n}, \qquad R_{04} = 0, \tag{B.2}$$

$$R_{mn} = -\frac{1}{2}\left(\phi_{,mn} + \psi_{,mn}\right), \tag{B.3}$$

$$R_{4n} = -\frac{1}{2}\phi_{,4n}, \tag{B.4}$$

$$R_{44} = \frac{1}{2}\left(\nabla^2\psi - \phi_{,44} - \psi_{,00}\right). \tag{B.5}$$

The Ricci scalar is

$$R = \nabla^2\phi + \nabla^2\psi - \phi_{,44} - \psi_{,00}. \tag{B.6}$$

The mixed Ricci tensor is given by

$$R^0_0 = \frac{1}{2}\left(\nabla^2\phi - \phi_{,44} - \psi_{,00}\right), \tag{B.7}$$

$$R^n_0 = \frac{1}{2}\psi_{,0n}, \qquad R^0_n = -\frac{1}{2}\psi_{,0n}, \tag{B.8}$$

$$R^4_0 = R^0_4 = 0, \tag{B.9}$$

$$R^n_m = \frac{1}{2}\left(\phi_{,mn} + \psi_{,mn}\right), \tag{B.10}$$

$$R^4_n = -\frac{1}{2}\phi_{,n4}, \qquad R^n_4 = \frac{1}{2}\phi_{,n4}, \tag{B.11}$$

$$R^4_4 = \frac{1}{2}\left(\nabla^2\psi - \phi_{,44} - \psi_{,00}\right). \tag{B.12}$$

Appendix C: Integration of the Universe Expansion Equation

The universe expansion was shown to be given by Eq. (3.16),

$$\frac{dr}{dv} = \tau \left[1 + (1 - \Omega_M)\, r^2/c^2\tau^2\right]^{1/2}.$$

This equation can be integrated exactly by the substitutions

$$\sin\chi = \alpha r/c\tau; \qquad \Omega_M > 1 \tag{C.1a}$$

$$\sinh\chi = \beta r/c\tau; \qquad \Omega_M < 1 \tag{C.1b}$$

where

$$\alpha = (\Omega_M - 1)^{1/2}, \qquad \beta = (1 - \Omega_M)^{1/2}. \tag{C.2}$$

For the $\Omega_M > 1$ case a straightforward calculation using Eq. (C.1a) gives

$$dr = (c\tau/\alpha)\cos\chi d\chi \tag{C.3}$$

and the equation of the universe expansion (3.16) yields

$$d\chi = (\alpha/c)\, dv. \tag{C.4a}$$

The integration of this equation gives

$$\chi = (\alpha/c)\, v + \text{const.} \tag{C.5a}$$

The constant can be determined using Eq. (C.1a). At $\chi = 0$, we have $r = 0$ and $v = 0$, thus

$$\chi = (\alpha/c)\, v, \tag{C.6a}$$

or, in terms of the distance, using (C.1a) again,

$$r\,(v) = (c\tau/\alpha)\sin\alpha v/c; \qquad \alpha = (\Omega_M - 1)^{1/2}. \tag{C.7a}$$

This is obviously a decelerating expansion.

For $\Omega_M < 1$, using Eq. (C.1b), a similar calculation yields for the universe expansion (3.16)

$$d\chi = (\beta/c)\, dv, \tag{C.4b}$$

thus

$$\chi = (\beta/c)\, v + \text{const.} \tag{C.5b}$$

Using the same initial conditions as above then gives

$$\chi = (\beta/c)\, v \tag{C.6b}$$

and in terms of distances,

$$r\,(v) = (c\tau/\beta)\sinh\beta v/c; \qquad \beta = (1-\Omega_M)^{1/2}\,. \tag{C.7b}$$

This is now an accelerating expansion.

For $\Omega_M = 1$ we have, from Eq. (3.16),

$$d^2r/dv^2 = 0. \tag{C.4c}$$

The solution is, of course,

$$r\,(v) = \tau v. \tag{C.7c}$$

This is a constant expansion.

Acknowledgements

It is a pleasure to thank Julia Goldbaum for calculating all needed geometrical expressions appearing in the paper, and Tanya Kuzmenko for useful discussions.

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